# Nash Bargaining Solution to Multi-Objective Resource Allocation and Sharing in SDC

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***Abstract*—Resource allocation and sharing among domains is a basic problem in the SDC for coalition partners. We propose a multi-objective framework for resource sharing and allocation among the SDC domains. To address the resource scarcity problem, we exploit resource sharing among different domains where each domain may have a particular objective (utility) to optimize. This results in a multi-objective optimization (MOO) problem for which we present an $N$-person Nash Bargaining Solution (NBS) to guarantee the Pareto optimality. To reduce the complexity of the N-person NBS, we transform it into an equivalent optimization problem that can be readily solved. We further prove that the strong duality property holds for the formulated problem, which thus enables us to develop a distributed algorithm for obtaining the NBS. Our experimental results confirm that besides improving the ability to satisfy the application demands for resources, the proposed NBS framework for resource allocation and sharing also improves the utility of different domains in the SDC.**

## I. Introduction

### A. Motivation

An SDC slice consists of multiple domains (enclaves) that have different types of resources. Applications can use these resources for a price, while domains are expected to provide resources according to a *Service Level Agreement* (SLA). However, demands for resources may exceed the capacities at individual domains. It is likely that resources of one domain may be over-utilized, while other domain's resources may be under-utilized. We propose resource sharing among such domains in the SDC to improve resource utilization and request satisfaction.

### B. Methodology and contributions

We consider a number of domains in the SDC slice which share their resources and form a logical resource pool to satisfy resource requests of different applications. We formulate such resource sharing and allocation among domains as a multi-objective optimization (MOO) problem, for which our goal is to achieve the Pareto optimal solution. Furthermore, since the Pareto solutions are spread over the *Pareto frontier* [1], choosing a single solution among them is challenging. As the *Nash Bargaining Solution* (NBS) [2] guarantees the provision of a fair and Pareto optimal solution to the MOO problem, we develop an NBS-based framework for resource allocation and sharing here. *To the best of our knowledge, this is the first generic NBS-based framework for resource sharing and allocation among domains.*

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## II. System Model

Let $\mathcal{N} = \{1, 2, \cdots, N\}$ be the set of all domains. We assume that each domain has a set of $\mathcal{K} = \{1, 2, \cdots, K\}$ types of resources such as communication, computation and storage resources. $C_n = \{C_{n,1}, \cdots, C_{n,K}\}$, with $C_{n,k}$ denoting the amount of type $k$ resources available at domain $n$. Each domain $n$ has a set of native applications $\mathcal{M}_n = \{1, 2, \cdots, M_n\}$. The set of all applications that request resources from all domains is given by $\mathcal{M} = \mathcal{M}_1 \cup \mathcal{M}_2 \cdots \cup \mathcal{M}_N$, where we assume $\mathcal{M}_i \cap \mathcal{M}_j = \emptyset, \ \forall i \neq j$, i.e., each application initially demands resources from only one and its native domain. $r^j_{n,k}$ is the amount of resource $k$ that application $j \in \mathcal{M}_n$ requests from domain $n$ (which is treated as a game player). When a domain is working alone (i.e., no resource sharing among domains), its objective is to maximize its utility by allocating resources to its native applications. A domain $n$ earns a utility $u^j_n(x^j_{n,k})$[1] by allocating $x^j_{n,k}$ amount of resource $k$ to application $j \in \mathcal{M}_n$, where the vector $\mathbf{x}^j_n = [x^j_{n,1}, x^j_{n,2}, \cdots, x^j_{n,K}]^T$. Below we present the optimization formulation for a single domain.

### A. Problem Formulation for Single Domain

For a single domain $n \in \mathcal{N}$, the allocation decision consists of vectors $\mathbf{x}^1_n, \cdots, \mathbf{x}^{M_n}_n$. The optimization problem is:

$$\max_{\mathbf{x}^1_n, \cdots, \mathbf{x}^{M_n}_n} \quad \sum_{j \in \mathcal{M}_n} u^j_n(\mathbf{x}^j_n), \tag{1a}$$

$$\text{s.t.} \quad \sum_j x^j_{n,k} \leq C_{n,k}, \quad \forall k \in \mathcal{K}, \tag{1b}$$

$$x^j_{n,k} \leq r^j_{n,k}, \quad \forall\, j \in \mathcal{M}_n, k \in \mathcal{K}, \tag{1c}$$

$$x^j_{n,k} \geq 0, \quad \forall\, j \in \mathcal{M}_n, k \in \mathcal{K}. \tag{1d}$$

### B. Problem Formulation for Multiple Domains

Let $C^j_m(x^j_{m,k})$ denote the communication cost for serving application $j \in \mathcal{M}_n$ to use resource at domain $m$ rather than its native domain $n$. By sharing its resources, domain

[1]We assume that each utility is a concave injective function for which the inverse of the first derivative exists, such as $(1 - e^{-x})$. Strictly speaking, our centralized NBS framework requires only concave injective utility function. However, the existence of the inverse of the first derivative is required for the distributed NBS.

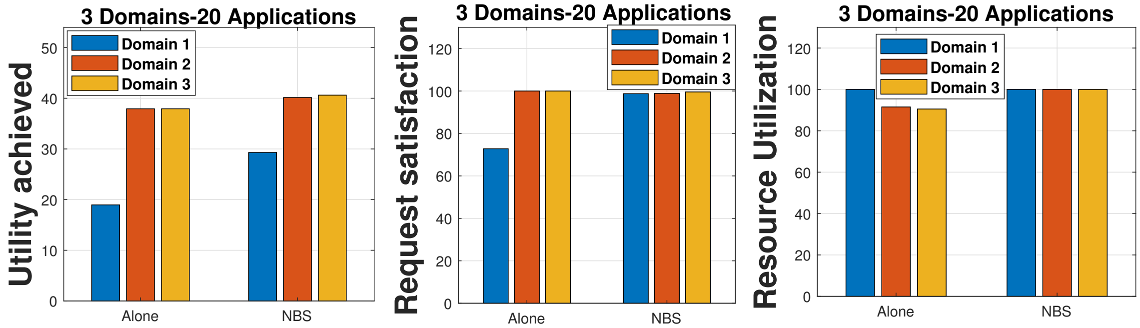


Fig. 1: Utility, average request satisfaction and average resource utilization for synthetic data with 3 Domains-20 Applications (per domain) when domains are working alone and using our proposed NBS framework.

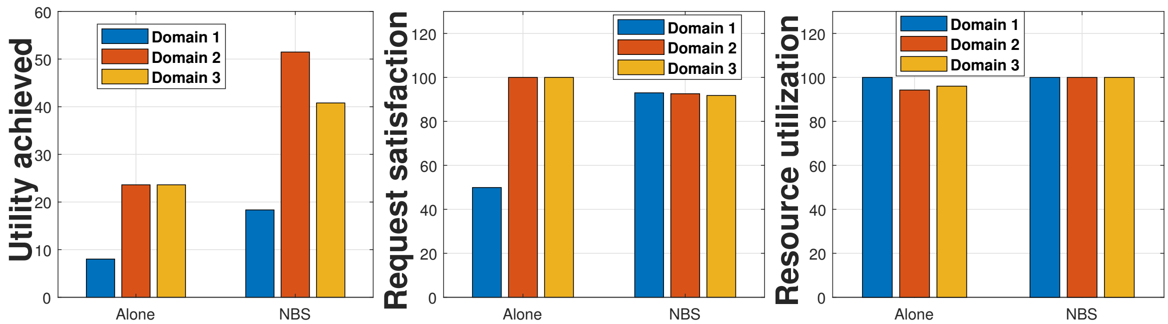


Fig. 2: Utility, average request satisfaction and average resource utilization for trace-files with 3 domains and 20 Applications when domains are working alone and using our proposed NBS framework.

$m$ earns a utility $u^j_m(x^j_{m,k}) - C^j_m(x^j_{m,k})$ after allocating $x^j_{m,k}$ for application $j \in \mathcal{M}_n$, where we assume that $u^j_n(x^j_{n,k}) = u^j_m(x^j_{m,k})$ when $x^j_{n,k} = x^j_{m,k}$. We also assume that $\mathbf{x}^j = [\sum_{n\in\mathcal{N}} x^j_{n,1}, \sum_{n\in\mathcal{N}} x^j_{n,2}, \cdots, \sum_{n\in\mathcal{N}} x^j_{n,K}]^T$, i.e., the total resource allocated to any application $j \in \mathcal{M}$ is the sum of resources allocated to it from all domains. The resource sharing and allocation framework, based on resource requests and capacities of domains, has to make an allocation decision that optimizes the utilities of all domains $n \in \mathcal{N}$ and satisfy user requests as well. The allocation decision is given by $\mathbf{X} = \{\mathbf{X}_1, \mathbf{X}_2, \cdots, \mathbf{X}_N\}$. Each provider $n \in \mathcal{N}$ solves the following multi-objective optimization problem.

$$\max_{\mathbf{x}_n} \sum_{j\in\mathcal{M}_n} u^j_n\Big(\sum_{m\in\mathcal{N}\setminus n} \mathbf{x}^j_m + \mathbf{x}^j_n\Big) + \Big(\sum_{l\in\{\mathcal{M}\setminus\mathcal{M}_n\}} u^l_n\Big(\sum_{m\in\mathcal{N}\setminus n} \mathbf{x}^l_m + \mathbf{x}^l_n\Big) - C^l_n(\mathbf{x}^l_n)\Big), \quad (2a)$$

$$\text{s.t.} \quad \sum_j x^j_{n,k} \le C_{n,k}, \quad \forall\, j \in \mathcal{M}, k \in \mathcal{K}, n \in \mathcal{N}, \quad (2b)$$

$$\sum_{m\in\mathcal{N}} x^j_{m,k} \le r^j_{n,k}, \quad \forall\, j \in \mathcal{M}, k \in \mathcal{K}, n \in \mathcal{N}, \quad (2c)$$

$$x^j_{n,k} \ge 0, \quad \forall\, j \in \mathcal{M}, k \in \mathcal{K}, n \in \mathcal{N}. \quad (2d)$$

## III. NBS for Resource Sharing among CSPs

The NBS for the MOO optimization problem in (2) can be obtained by solving the following optimization problem:

$$\max_{\mathbf{X}} \prod_{n=1}^{N} \Bigg( \sum_{j\in\mathcal{M}_n} u^j_n\Big(\sum_{m\in\mathcal{N}\setminus n} \mathbf{x}^j_m + \mathbf{x}^j_n\Big) + \Big(\sum_{l\in\{\mathcal{M}\setminus\mathcal{M}_n\}} u^l_n\Big(\sum_{m\in\mathcal{N}\setminus n} \mathbf{x}^l_m + \mathbf{x}^l_n\Big) - C^l_n(\mathbf{x}^l_n)\Big) - d^0_n \Bigg), \quad (3a)$$

$$\text{s.t.} \quad Constraints\ in\ (2b) - (2d),$$
$$\sum_{j\in\mathcal{M}_n} u^j_n\Big(\sum_{m\in\mathcal{N}\setminus n} \mathbf{x}^j_m + \mathbf{x}^j_n\Big) + \Big(\sum_{l\in\{\mathcal{M}\setminus\mathcal{M}_n\}} u^l_n\Big(\sum_{m\in\mathcal{N}\setminus n} \mathbf{x}^l_m + \mathbf{x}^l_n\Big) - C^l_n(\mathbf{x}^l_n)\Big) > d^0_n, \quad \forall\, n \in \mathcal{N}. \quad (3b)$$

We propose a central algorithm as well as an *iterative, distributed algorithm* for NBS. To enable development of the distributed algorithm, we transform the problem into an equivalent one [3] and prove the strong duality property holds. Figures 1 and 2 show that our proposed framework improves the utilities of domains, request satisfaction and resource utilization.

## References

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