Competitive Influence Maximisation using Voting Dynamics

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***Abstract*—While there exists considerable work that looks into optimally influencing individuals within a given network to maximise the overall spread of influence, existing work in this area typically neglects a few key features present in realistic application domains: (i) individuals may change their opinions dynamically over time, (ii) varying amounts of resources (or incentives) may be expended on individuals to influence their opinions. In this paper, we address these shortcomings. Specifically, using the principles of game theory and voter dynamics, we propose a computational model that solves this optimisation problem of allocating continuous resources within a given budget to maximise influence spread in the presence of an adversary (whose strategy may be known or unknown) in a canonical star topology.**

I. Introduction

Individual opinions and behaviours are largely affected by peer influence [3]. The propagation of such influence, through societies, predominantly depend on the exchange of information between individuals through their social connections [4]. This inherent connectivity in social networks, can be exploited to fuel several agendas, where societies are externally manipulated through information control, to dominate aggregate social behaviours or steer public opinions, with both positive [8] and negative [**?**] intent. However, as an external agent trying to influence a social network, the most challenging task, which is the crux of what is known as the *Influence Maximisation (IM)* problem [1], is to determine the most optimal distribution of resources that maximises the influence spread.

Here, we identify such optimal strategies for maximising influence within a social network in competitive settings under budget constraints. While existing work has focussed on simple threshold models, we consider more realistic settings, where (i) states are dynamic, i.e., nodes oscillate between influenced and uninfluenced states, and (ii) continuous amounts of resources (e.g., incentives or effort) can be expended on the nodes.

We propose a mathematical model using voting dynamics to characterise optimal strategies in a prototypical star topology against known and unknown adversarial strategies. In cases where the adversarial strategy is unknown, we characterise the Nash Equilibrium. To generalise the work further, we introduce a fixed cost incurred to gain access to nodes, together with the dynamic cost proportional to the influence exerted on the nodes, constrained by the same budget. We observe that, as the cost changes, the system interpolates between the historic discrete and the current continuous case.

II. Model

We represent social networks as graphs $G(V, E)$ of vertices (V) and edges (E), where vertices (V) indicate social agents and edges (E), the relationships between them. The strength of these connections are given by the weight adjacency matrix *W* of the graph *G*, where the ij^{th} entry w_{ij}, gives the strength of the influence, an agent j has on i. Here, agents $i, j \in V$ and diagonal elements $w_{ii} = 0$ for all agents $\forall i \in V$.

We assume that *two* external influencers, *A* and *B* compete to maximise their influence in the network. Influence propagation in this model follows voting dynamics [7], where at each update event, a node i is selected at random (with probability $\frac{1}{|V|}$), which then either adopts the state of a randomly (with probability $\frac{1}{deg(i)}$) chosen neighbour j or that of an external influencer *A* or *B*, when directly influenced.

The probabilistic state of a node i is given by $u_{A,i}$. At equilibrium, the fraction of the total nodes in any one particular state, say $\sigma_i = A$, is given by $u_{A,avg} = \frac{\sum_i^N u_{A,i}}{N}$. When represented as a non-cooperative game, each player here is driven by the intent to increase their individual *pay-off*s $u_{A,avg} = \frac{\sum_i^N u_{A,i}}{N}$ and $u_{B,avg} = \frac{\sum_i^N u_{B,i}}{N}$.

For any arbitrary network of *N* nodes, the rate at which an influencer *A* spreads its influence in the network, is given by:

$$\frac{du_{A,i}}{dt} = (1 - u_{A,i}).pr_i(A) - u_{A,i}.pr_i(B). \qquad (1)$$

where $pr_i(A)$ is the probability with which a node i adopts state *A* and $pr_i(B)$ the probability with which it adopts state *B*. Note that, the amounts of influence $p_{A,i}$ and $p_{B,i}$, exerted on any node i here is continuous and proportional to the amount of resource expended on it. Hence, we use resource and influence inter-changeably throughout the course of the paper.

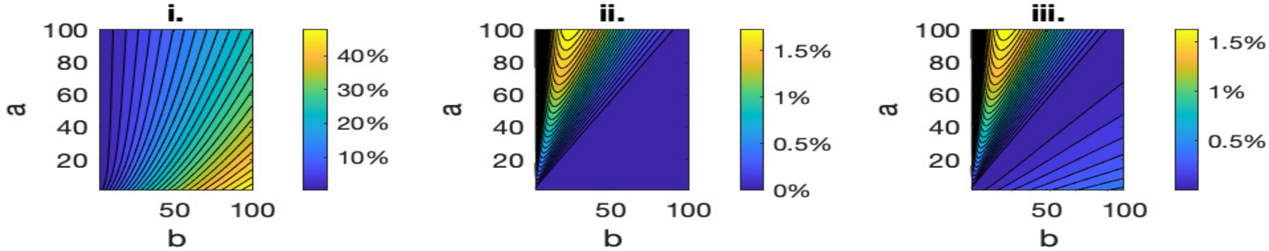

Fig. 1: Heatmaps showing the % gain in $u_{A,avg}$, when A plays the optimal strategy, in comparison to (i) targeting the hub node, (ii) targeting the periphery and (iii) targeting all nodes uniformly, against adversary B who targets the hub node, for different values of available resources a and b.

When the system reaches steady-state, although individual states are still dynamic and change with time, the total number nodes in any one particular state in the network, no longer change with time and therefore $\frac{du_{A,i}}{dt} = 0$, gives us

$$\left[L + diag(\boldsymbol{p_A} + \boldsymbol{p_B})\right] \boldsymbol{u_A} = \boldsymbol{p_A}. \tag{2}$$

Here *L*, the Laplacian of the network is given by the *N-by-N* matrix, where each element $L_{ij} = \delta_{ij} \sum_k w_{ki} - (1 - \delta_{ij}) w_{ji}$, and δ_{ij} is the *kronecker delta*.

The objective of this problem is now simply to determine the best possible allocation of resources $\boldsymbol{p_A}$, given a fixed a, that maximises average opinion $u_{A,avg}$ in the network. We explore this problem for multiple adversarial settings in star networks. In principle, this method can be extended to other simple networks as well, such as, complete graphs, line graphs, cycles and even small scale-free networks, for which L can be expressed analytically. For arbitrary weighted graphs, the optimisation problem can be solved numerically using 2, given a fixed budget, a and b.

III. Results

We find that, in case of known adversarial strategies, the best response strategy, when competing influence on the network exceeds available budget, is to target nodes avoided by the adversary with more influence. As available budget increases, more influence is expended on nodes targeted by the adversary. Where adversarial strategy is unknown, we show that the pure-strategy *Nash Equilibrium* is when both competitors target all nodes equally. To generalise the work further, we introduce a fixed cost incurred to gain access to nodes, together with the dynamic cost proportional to the influence exerted on the nodes, constrained by the same budget. We also observe that the optimal strategies change when a fixed cost c is incurred to gain access to nodes in the network. As c increases, the system interpolates between the continuous and discrete models. Where the adversary targets the hub, we find that with increase in cost c, the optimal strategy switches between targeting all nodes, to targeting only the hub node very abruptly. In all other cases of known adversarial strategies, we observe a less abrupt shift in the optimal strategy.

For detailed results please refer to: *Chakraborty S, Stein S, Brede M, Swami A, DeMel G, Restocchi V. Competitive Influence Maximisation using Voting Dynamics. In: The Social Influence Work- shop (5th edition) at the 2019 IEEE/ACM International Conference on Advances in Social Networks Analysis and Mining, Vancouver, Canada..*

IV. Conclusion

We observe that as adversarial influence increases on the network, the optimal strategy is to shift resources from nodes targeted by the adversary to those not targeted by the adversary. Another key finding of this research, in contrast to earlier work [5] [2] demonstrates the importance of low-degree nodes over high-degree nodes.

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